Newton Raphson Method

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Abstract -- The paper is about Newton Raphson Method which is all-inclusive to solve the non-square and non-linear problems. The study also aims to comparing the rate of performance, rate of convergence of Bisection method, root findings of the Newton meted and Secant method. It also represents a new approach of calculation using nonlinear equation and this will be similar to Newton Raphson simple method and inverse Jacobian matrix will be used for the iteration process and this will be further used for distributed power load flow calculation and will also be helpful in some of the applications. The paper also discusses the difference between the use of built in derivative function and self-derivative function in solving non-linear equation in scientific calculator. The derivation Newton Raphson formula, algorithm, use and drawbacks of Newton Raphson Method have also been discussed.

Index Terms – Homotopy method, complex methods, bracketing method, convergence method, iteration method, self-derivation, algorithm complexity, square root of 2, computational.

1 Introduction

Finding the solution to the set of nonlinear equations $f(x) = (f1,.......f^n)' = 0$ is been a problem for the past years. Here we consider this nonlinear equation and try find the solution to it and this can be found out by the Newton Raphson method. This method is very familiar for its fast rate convergence and for improving the convergence property, the Homotopy method is adopted out of various methods. Homotopy works by transforming an original problem into an easy problem which will be afterwards become easy to be solved. A Homotopy map will also be required in further solving the problem.

Root finding is also one of the problems in practical applications. Newton method is very fast and efficient as compared to the others methods. In order to compare the performance, it is therefore very important to observe the cost and speed of the convergence. Newton method requires only one iteration and the derivative evaluation per iteration. The result of comparing the rate of convergence of Bisection, Newton and Secant methods came as Bisection method < Newton method < Secant method which in terms of number is that the Newton method is 7.678622465 times better than the Bisection method whereas Secant method is 1.389482397 times better than the newton method.

Complex systems with higher speed processing control are in demand now a days and the solution to this is to divide them into subsystems and in this way each subsystem will be treated individually and the control and operation will be applied to each of that subsystem. A new technique of distributed load flow calculation using nonlinear is also presented in the paper.

Finding roots of the nonlinear equation with the help of Newton Raphson method provides good result with fast convergence speed and Mat lab also adopted this method for finding the roots and tool used for such calculations is scientific calculator.

Bracketing method is which requires bracketing of the root by two guesses are always convergent as they are based on reducing the interval between two guesses. Bisection method and the false position method makes use of the bracketing method.

2 Newton Raphson Method

2.1 Definition

Newton's method (also acknowledged as the Newton–Raphson method), named after Isaac Newton and Joseph Raphson, is a technique for judgment sequentially superior approximations to the extraction (or zeroes) of a real-valued function.

$$x : f(x) = 0 .$$

Any zero-finding method (Bisection Method, False Position Method, Newton-Raphson, etc.) can also be used to find a minimum or maximum of such a function, by finding a zero in the function's first derivative, see Newton's method as an optimization algorithm.

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2.2 Explanation

The idea of the Newton-Raphson method is as follows: one starts with an preliminary conjecture which is logically secure to the true root, then the purpose is approximated by its digression line (which can be computed using the tools of calculus), and one computes the x-intercept of this digression line (which is effortlessly done with simple algebra). This x-intercept will typically be a enhanced approximation to the function's root than the original guess, and the method can be iterated Based on collinear scaling and local quadratic approximation, quasi-Newton methods have improved for function value is not fully used in the Hessian matrix. As collinear scaling factor in paper may appear singular, this paper, a new collinear scaling factor is studied. Using local quadratic approximation, an improved collinear scaling algorithm to strengthen the stability is presented, and the global convergence of the algorithm is proved. In addition, numerical results of training neural network with the improved collinear scaling algorithm shown the efficiency of this algorithm is much better than traditional one.

2.3 Derivation

In numerical analysis, **Newton's method** (also known as the **Newton–Raphson method**), named after Isaac Newton and Joseph Raphson, is a method for finding successively better approximations to the roots (or zeroes) of a real-valued function.

$$x : f(x) = 0 \,.$$

The Newton–Raphson method in one variable is implemented as follows:

Given a function f defined over the reals x, and its derivativef', we begin with a first guess x_0 for a root of the function f. Provided the function satisfies all the assumptions made in the derivation of the formula, a better approximation x_1 is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \,.$$

Geometrically, $(x_1, 0)$ is the intersection with the x-axis of the tangent to the graphof f at $(x_0, f(x_0))$.

The process is repeated as

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Until a sufficiently accurate value is reached.

2.4 Examples

Square root of a number

Consider the problem of finding the square root of a number. Newton's method is one of many methods of computing square roots.

For example, if one wishes to find the square root of 612, this is equivalent to finding the solution to

$$x^2 = 612$$

The function to use in Newton's method is then,

$$f(x) = x^2 - 612$$

With derivative,

$$f'(x) = 2x.$$

With an initial guess of 10, the sequence given by Newton's method is

$$\begin{aligned}
x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} &= 10 - \frac{10^2 - 612}{2 \cdot 10} &= 35.6 \\
x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} &= 35.6 - \frac{35.6^2 - 612}{2 \cdot 35.6} &= \underline{2}6.395505617978\ldots \\
x_3 &= \vdots &= \vdots &= \underline{24.7}90635492455\ldots \\
x_4 &= \vdots &= \vdots &= \underline{24.738688}294075\ldots \\
x_5 &= \vdots &= \vdots &= \underline{24.7386337537}67\ldots
\end{aligned}$$

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Where the correct digits are underlined. With only a few iterations one can obtain a solution accurate to many decimal places.

Solution of cos(x) = x^3

Consider the problem of finding the positive number x with cos(x) = x^3. We can rephrase that as finding the zero of (x) = cos(x) − x^3. We have $f'(x) = -\sin(x) - 3x^2$. Since cos(x) ≤ 1 for all x and x^3> 1 for x > 1, we know that our solution lies between 0 and 1. We try a starting value of x_0 = 0.5. (Note that a starting value of 0 will lead to an undefined result, showing the importance of using a starting point that is close to the solution.)

$$
\begin{array}{lclclcl}
x_1 & = & x_0 - \dfrac{f(x_0)}{f'(x_0)} & = & 0.5 - \dfrac{\cos(0.5) - (0.5)^3}{-\sin(0.5) - 3(0.5)^2} & = & 1.11 \\
x_2 & = & x_1 - \dfrac{f(x_1)}{f'(x_1)} & = & \vdots & = & \underline{0.9}0 \\
x_3 & = & \vdots & = & \vdots & = & \underline{0.86} \\
x_4 & = & \vdots & = & \vdots & = & \underline{0.86} \\
x_5 & = & \vdots & = & \vdots & = & \underline{0.86} \\
x_6 & = & \vdots & = & \vdots & = & \underline{0.86}
\end{array}
$$

The correct digits are underlined in the above example. In particular, x_6 is correct to the number of decimal places given. We see that the number of correct digits after the decimal point increases from 2 (for x_3) to 5 and 10, illustrating the quadratic convergence.

Example:

Given

$$f(x) = 2x^{3.4} + 4x^2 + x - 8 = 0$$

, one may re-write it as:

$$x = g(x) = 8 - 2x^{3.4} - 4x^2$$

$$x = g(x) = \frac{\sqrt{8 - x - 2x^{3.4}}}{2}$$

or,

or,

$$x = g(x) = \{(8 - x - 4x^2)/2\}^{1/3.4}$$

Where g(x) denotes possible choice iteration function.

Example: By using the Newton-Raphson's method find the positive root of the quadratic equation

$5x^2 + 11x$ Correct to 3 significant figures.

$$f(x) = 5x^2 + 11x - 17$$

$$f'(x) = 10x + 11$$

Choose $x_1 = 1$,

$$f(x_1) = f(1) = 5 \cdot (1)^2 + 11 \cdot (1) - 17 = 5 + 11 - 17 = -1$$

$$f' \cdot (x_1) = f'(1) = 10 \cdot (1) + 11 = 10 + 11 = 21$$

$$x_2 = x_1 - \frac{f(x_1)}{f' \cdot (x_1)} = 1 - \frac{f(1)}{f'(1)} = 1 - \left(-\frac{1}{21}\right) = 1 + \frac{1}{21} = 1.047619048$$

$x_2 = 1.047619048$,

$$f' \cdot (x_2) = f'(1.047619048) = 10 \cdot (1.047619048) + 11 = 10.47619048 + 11 = 21.47619048$$

Checking it by the quadratic formula

$$x = \frac{-11 \pm \sqrt{11^2 - 4 \cdot 5 \cdot (-17)}}{2 \cdot 5}$$

$$x = \frac{-11 \pm \sqrt{121 + 340}}{10}$$

$$x = \frac{-11 \pm \sqrt{461}}{10}$$

Use the + to get the positive root:

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x=1.047091055

So we only needed 1 iteration of the Newton-Raphson method to get it to three significant figures, for what we had then would have rounded to 1.05. By taking it one more step we have it to 4 significant figures 1.047[2].

3 Convergence of Newton-Raphson method:

Suppose x_r is a root of $f(x)=0$ and x_n is an estimate of x_r s.t. $|x_r - x_n| = \delta << 1$.
Then by Taylor series expansion we have,

$$0 = f(x_r) = f(x_n+\delta) = f(x_n) + f'(x_n)(x_r - x_n) + \frac{f''(\xi)}{2}(x_r - x_n)^2 \qquad (1^*)$$

For some ξ between x_n and x_r.
By Newton-Raphson method, we know that

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

i.e.

$$f(x_n) = f'(x_n)(x_n - x_{n+1}) \qquad (2^*)$$

Using (2*) in (1*) we get

$$0 = f'(x_n)(x_r - x_{n+1}) + \frac{f''(\xi)}{2}(x_r - x_n)^2$$

Say

$$e_n = (x_r - x_n), \quad e_{n+1} = x_r - x_{n+1}$$

Where $e_n, \; e_{n+1}$ denote the error in the solution at n th and (n+1) th iterations.

$$\therefore \quad e_{n+1} = -\frac{f''(\xi)}{2f'(x_n)} \sim e_n^2$$

$$\Rightarrow e_{n+1} \propto e_n^2$$

∴ Newton Raphson Method is said to have quadraticconvergence.

Note:
Alternatively, one can also prove the quadratic convergence of Newton-Raphson method based on the fixed - point theory. It is worth stating few comments on this approach as it is a more general approach covering most of the iteration schemes discussed earlier.

A Brief discussion on Fixed Point Iteration:
Suppose that we are given a function

$$f(x) = 0 \qquad (i)$$

on an interval $[a,b]$ for which we need to find a root. Derive, from it, an equation of the form:

$$x = g(x) \qquad (ii)$$

Any solution to (ii) is called a fixed point and it is a solution of (i). The function g(x) is called as "Iteration function".

4 Conclusion

From the referenced research papers we have concluded that the ,The convergence rate of Newton method is fast as compared to other methods .However the current injection method has simple Jacobian matrix and smaller computation in every iteration, which can make the programming easier and reduce the time of the computation Secant method is the most effective method it has a converging rate close to that of the Newton Raphson method but it requires only a single function evaluating per iteration. We have also researched that the convergence rate of bisection method is very slow and it's difficult to extend such kind of systems equations. So in comparison Newton method have a fast converging rate.

The effectiveness of using scientific calculator in solving non-linear equations using Newton-Raphson method also reduces the time complexity for solving nonlinear equations. With the help of the built in derivative functions in calculator now we can able to calculate the nonlinear functions faster. This research also shows that the common mistakes made by the participants had been reduced after they were taught the technique to solve the problem using the calculator. In this work a sequence of i t e r a t i v e methods for solving nonlinear equation f(x) = 0 with higher-order convergence is developed. The method can be continuously applied to generate an iterative scheme with arbitrarily specified order of convergence.

We also concluded that the Newton Raphosn method can be used very effectively to determine the intrinsic value based on its measured permittivity.

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